2011-2012
IB MYP YEAR 4

SUMMATIVE ASSESSMENT

Year 9 Mathematics (Extended) Teacher: **Ms Li**, **Mr Millard** and **Mr So**

Name: Suggested solution [9]

Date of task: **8th June, 2012** Time allowed: **1.5 hours**

Student's **Performance** in Different **Criteria**			
A		**C**	

Instructions

- Read the instructions for all questions carefully.
- All work must be hand written.
- All work, steps and proper units must be shown.
- A non-electronic dictionary is allowed.
- Use of calculator is allowed.

Advice:

- Read the criteria descriptors and task-specific rubrics carefully before you start your work. This will give you a clear understanding of what is required and what a high quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest levels in this task.

- This assessment task will be assessed on Criterion **A & C**.
  - For Criteria **A**, the questions are all assigned with levels;
  - Criterion **C** will be assessed as an overall impression on the presentation of work in this assessment.

ASSESSMENT CRITERIA

Criterion A: KNOWLEDGE AND UNDERSTANDING

Achievement level	Task Specific Rubric	IBO Published Descriptor
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.
1–2	The student can solve **some** **simple** problems.	The student **attempts** to make deductions when solving **simple** problems in **familiar** contexts.
3–4	The student can solve **most** **simple** and **some** **more complex** problems.	The student **sometimes** makes **appropriate** deductions when solving **simple and more-complex** problems in **familiar** contexts
5–6	The student can solve **some** **challenging** problem along with **all** different types of problems.	The student **generally** makes **appropriate** deductions when solving **challenging** problems in a **variety** of **familiar** contexts.
7–8	The student can solve **most** challenging and **most** **unfamiliar** problems along with **all** different types of problems.	The student **consistently** makes **appropriate** deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar** situations.

Criterion C: COMMUNICATION IN MATHEMATICS

Achievement level	Task Specific Rubric	IBO Published Descriptor
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.
1–2	The student should be able to explain **some problems** step by step. The lines of reasoning are **difficult to follow**.	The student shows **basic** use of mathematical language **and/or** forms of mathematical representation. The lines of reasoning are **difficult to follow**.
3–4	The student should be able to explain **most problems** step by step. The lines of reasoning are **clear** though **not always** **logical** or **complete**.	The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **clear** though not always **logical** or **complete**. The student moves between different forms of representation **with some success**.
5–6	The student should be able to explain **most problems** step by step. The lines of reasoning are **concise**, **logical** and **complete**. The student use **correct unit** in the questions.	The student shows **good** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **concise**, **logical** and **complete**. The student moves **effectively** between different forms of representation.

A. SIMPLE PROBLEMS

Suggested time allocation for <u>Question **1 to 5** is **15 minutes**</u>.

1. Given the points A (–1 , 2) and B = (2 , **k**), find the value(s) of **k** such that the **<u>length of line AB is 5 units</u>**.

$$5 = \sqrt{(-1-2)^2 + (2-k)^2}$$

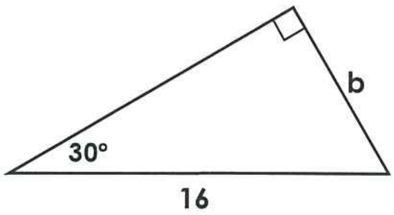
$$25 = 9 + 4 - 4k + k^2$$

$$k^2 - 4k - 12 = 0$$

$$(k-6)(k+2) = 0$$

$$k = 6 \text{ or } k = -2$$

2. In the figure on the right, find the value of **b** **<u>without using calculator</u>**.



$$\sin 30° = \frac{b}{16}$$

$$\frac{1}{2} = \frac{b}{16}$$

$$b = 8$$

3. Given that the equation of the line **L1** is $y - 2x = 4$, which of the following line(s) is/are **parallel to L1**? Which of the following line(s) has/have **negative y-intercepts**?

L2: $y = -2x + 4$ **L3:** $2y - 4x - 5 = 0$ **L4:** $-3y = 2x + 4$

<u>Explain your answers by showing your calculations.</u>

slope of L1 = 2, y-intercept of L1 = 4

slope of L2 = -2, y-intercept of L2 = 4

L3: $2y = 4x + 5$

$y = 2x + \frac{5}{2}$

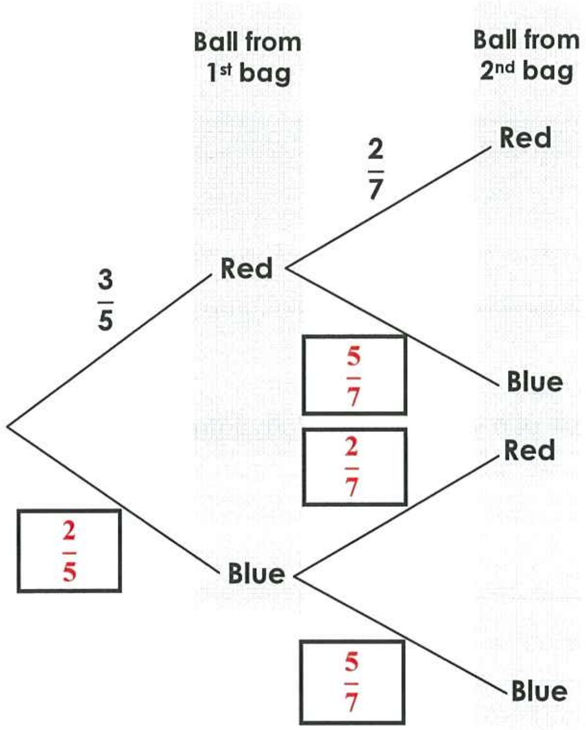
slope of L3 = 2, y-intercept of L3 = $\frac{5}{2}$

L4: $y = -\frac{2}{3}x - \frac{4}{3}$

slope of L4 = $-\frac{2}{3}$, *y-intercept of L4 =* $-\frac{4}{3}$

4. Loren has two bags. The **first** bag contains **3 red** balls and **2 blue** balls. The **second** bag contains **2 red** balls and **5 blue** balls. Loren takes **1 ball** at random from **each bag**.

(a) Complete the probability **tree diagram** by entering the **correct answers into the boxes.**



(b) Find the probability that Loren takes **two red balls.**

$$\frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$$

5. Evaluate the following **without using calculator.**

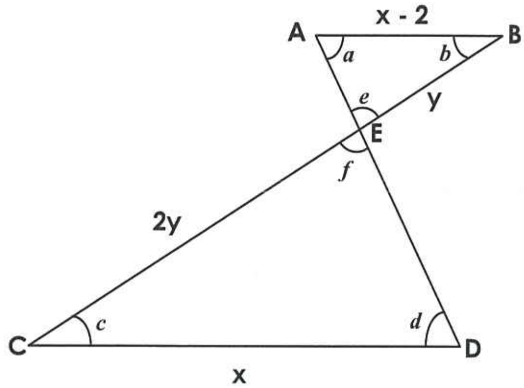
$$\sin^2 23° + \cos^2 23° - \frac{\sin 45°}{\cos 45°}$$

$(\sin^2 23° + \cos^2 23°) - \tan 45°$
$= 1 - 1$
$= 0$

B. MORE COMPLEX PROBLEMS

Suggested time allocation for <u>Question **6 to 9** is **25 minutes**</u>.

6. In the figure below, the line AB is parallel to the line CD and some dimensions are shown in terms of x or y.



(a) Show that Δ **ABE** and Δ**DCE** are **similar**. State the reason(s) if necessary.

$\angle b = \angle c$ (alt. $\angle$, $AB \parallel CD$)

$\angle a = \angle d$ (alt. $\angle$, $AB \parallel CD$)

$\angle e = \angle f$ (vert. opp. $\angle$s)

$\therefore \Delta ABE = \Delta DCE$ (A.A.A.)

(b) Find the value of **x**.

$\because \Delta ABE = \Delta DCE$

$$\therefore \frac{AB}{CD} = \frac{EB}{EC}$$

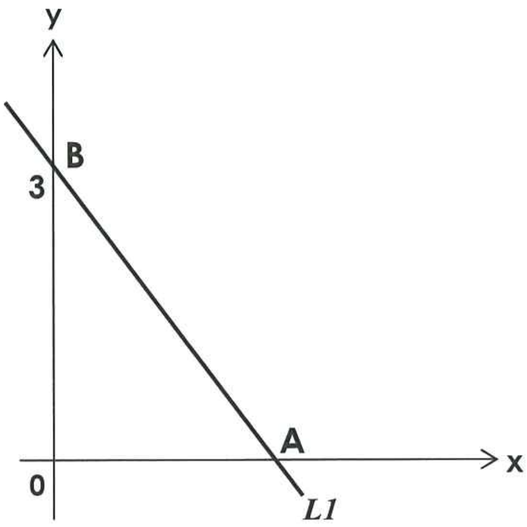
$$\frac{x-2}{x} = \frac{y}{2y}$$

$$\frac{x-2}{x} = \frac{1}{2}$$

$$2x - 4 = x$$

$$x = 4$$

7. In the graph on the right, a line $L1$ cuts the x-axis and y-axis at point **A** and **B** respectively. The y-intercept is 3.



(a) If the area of the triangle AOB is 3 square units, find the **equation** of $L1$. Express your answer in **slope-intercept form**.

$$\text{area of triangle } AOB = 3$$
$$\text{area of triangle } AOB = \frac{1}{2}\times(AO)(BO) = 3$$
$$\therefore\ \frac{1}{2}\times(AO)(3) = 3$$
$$AO = 2$$
$$\therefore \text{coordinates of } A = (2,0)$$
$$\frac{x}{2} + \frac{y}{3} = 1$$
$$\frac{3x+2y}{6} = 1$$
$$3x + 2y = 6$$
$$y = -\frac{3}{2}x + 3$$

(b) If a line $L2$ is **perpendicular** to $L1$ and two lines intersect at point **D(4,-3)**, find the equation of $L2$. Express your answer in **general form**.

$$\text{slope of } L1 = -\frac{3}{2}$$
$$\because\ L1 \perp L2$$
$$\therefore\ \text{slope of } L2 \times \text{slope of } L1 = -1$$
$$\text{slope of } L2 \times -\frac{3}{2} = -1$$
$$\text{slope of } L2 = \frac{2}{3}$$

Equation of L2:
$$\frac{y-(-3)}{x-4} = \frac{2}{3}$$
$$3y + 9 = 2x - 8$$
$$3y - 2x + 1 = 0$$
17

$y = \frac{2x-17}{3}$

$y = mx + c$

$y = \frac{2}{3}x - \frac{17}{3}$

$3y = 2x - 17$

$2x - 3y - 17 = 0$

8. In a certain dice game, the player throws **two** typical unbiased **six-faces dice** and receives **$5** if the sum is **7 or 11**, otherwise he or she **pays $2**.

(a) Calculate the probability of obtaining the **sum of 7 or 11** when you **<u>throw the two dice once</u>**.

No. of possible outcomes when two dice are thrown = 6× 6 = 36

Favorable outcomes when the sum is 7 are (1,6), (2,5), (3,4), (4,3), (5,2) and (6,1)

No. of favorable outcomes when the sum is 7 = 6

$$\therefore P(sum = 7) = \frac{6}{36} = \frac{1}{6}$$

Favorable outcomes when the sum is 11 are (5,6) and (6,5)

No. of favorable outcomes when the sum is 11 = 2

$$\therefore P(sum = 11) = \frac{2}{36} = \frac{1}{18}$$

$$\therefore P(sum = 7 \text{ or } 11) = \frac{1}{6} + \frac{1}{18} = \frac{4}{18} = \frac{2}{9}$$

(b) If you play the game **18 times**, calculate the **amount of money** you **<u>expect to gain or lose</u>**.

$$\text{Expected value} = \left[\left(5\times\frac{2}{9}\right) - \left(2\times\frac{7}{9}\right)\right]\times 18$$

$$= \left[\left(\frac{10}{9}\right) - \left(\frac{14}{9}\right)\right]\times 18$$

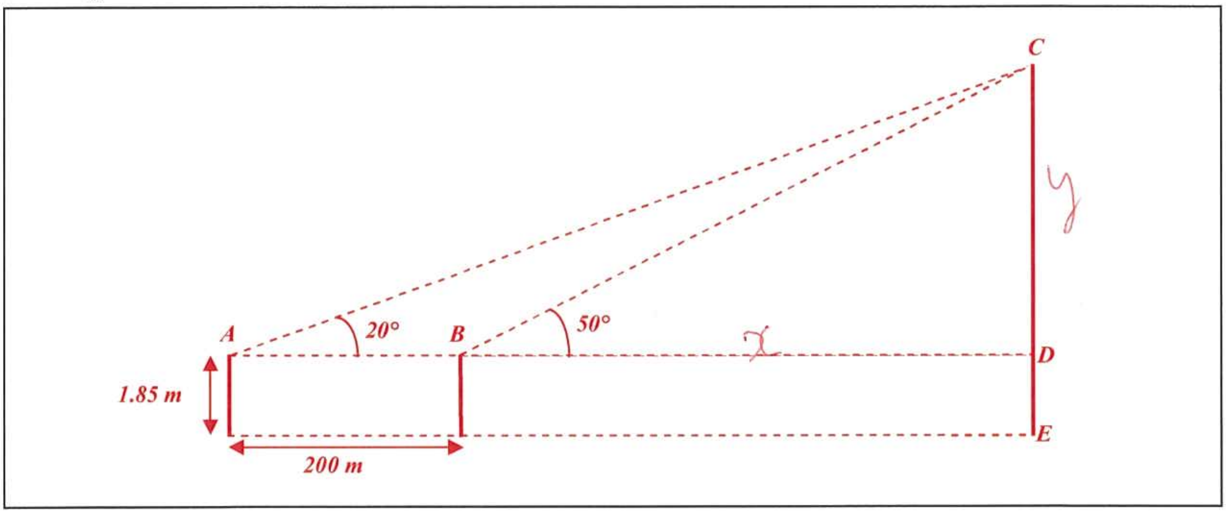
$$= \frac{-4}{9}\times 18$$

$$= \$-8$$

$\therefore$ *You expect to <u>lose $8</u> after played 18 times.*

9. Mr Bolivar, a volunteer fireman who is 1.85 m tall, is running towards a burning building where there is a fire on the roof. Initially, his angle of elevation to the roof is 20°. He runs for 200 m and now his angle of elevation is 50°. Assume that the ground is horizontal and the building is vertical.

(a) Sketch a **diagram** to represent the information above. You may add new labels on your drawing.



(b) How tall is the building? Correct your answer to the **nearest meter**.

Let (x)m be the distance of BD and (y)m be the height of CD.

Consider the triangle CDB, $\tan 50° = \dfrac{y}{x}$

$$y = x \tan 50°$$

Consider the triangle CDA, $\tan 20° = \dfrac{y}{x+200}$

$$y = (x+200) \tan 20°$$

$\therefore\ x \tan 50° = (x+200) \tan 20°$

$1.192x = (200 + x)(0.364)$

$1.192x = 72.794 + 0.364x$

$1.192x - 0.364x = 72.794$

$0.828\,x = 72.794$

$x = 87.92$

$= 88\ m$ (correct to nearest meter)

C. CHALLENGING PROBLEM

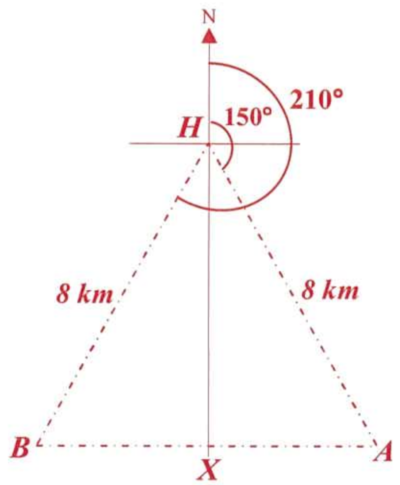
Suggested time allocation for Question **10 and 11** is **30 minutes**.

10. Ship **A** leaves the harbor H on a bearing **150°** with a speed of **40** km/hr. At the same time, Ship **B** leaves harbor H on a bearing **210°** with a speed of **40** km/hr.

(a) **After 12 minutes**, how far did ship A and ship B travel?

After 12 mins, ship A travelled 8 *km* (= $40 \times \frac{12}{60}$) *and ship* B *travelled* 8 *km* (= $40 \times \frac{12}{60}$).

(b) Sketch a **diagram** to represent the information above **12 minutes after the two ships left the harbor.**



(c) Find the **true bearing from Ship A to Ship B** 12 minutes after they left the harbor.

W0° or W *(Not accept* **"due West"** *or* **"West"** *or* **"270°")**

(d) Find the **distance between the two ships** 12 minutes after they left the harbor. Give your answer to the **nearest meter.**

Consider the triangle AHX, $\sin 30° = \frac{AX}{AH}$

$$\frac{1}{2} = \frac{AX}{8}$$

$$AX = 4$$

Consider the triangle BHX, $\sin 30° = \frac{BX}{BH}$

$$\frac{1}{2} = \frac{BX}{8}$$

$$BX = 4$$

$\therefore AB = AX + BX = 4 + 4 = \underline{\underline{8\ km}}$

11. The properties of a rectangle and a square are given below:

- ◆ The length of the rectangle is 3 cm longer than the side of the square.
- ◆ The width of the rectangle is double the length of the side of the square.

If the **sum of their areas** is **24 cm²**, find the **dimensions** (that is, its length and width) of the rectangle.

Let (x)cm be the length of the side of the square.

$\therefore$ *Length of the rectangle* = $(x + 3)$*cm*

Length of the rectangle = $(2x)$*cm*

Area of the square = x^2

Area of the rectangle = $(x + 3)(2x) = 2x^2 + 6x$

$\therefore$ *Sum of the areas of the square and the rectangle* = $x^2 + 2x^2 + 6x = 3x^2 + 6x$

Given that the sum of the areas is 24 cm²,

$\therefore\ 3x^2 + 6x = 24$

$3x^2 + 6x - 24 = 0$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4$ or $x = 2$

$\because$ *Length of the side of the square cannot be negative,*

$\therefore$ $x = -4$ *is rejected*

$\therefore\ x = 2$

$\therefore$ *Length of the rectangle* = $2 + 3$ = $\underline{\underline{5\ cm}}$

$\therefore$ *Width of the rectangle* = $2(2)$ = $\underline{\underline{4\ cm}}$

D. Unfamiliar problems (Suggested time allocation for Question **12 and 13** is **30 minutes**.)

12. At noon, Tom and Pete both park at the same starting point. Tom starts to ride his bike at 8 miles/hr. Two hours later, Pete starts after Tom on a bicycle at 12 miles/hr.

(a) How far will Tom have ridden before he is **overtaken by Pete**?

After 2 hours, Tom have ridden 16 miles (= 8 × 2).

Let (x)hours be the time when Tom is overtaken by Pete.

$\therefore\ 8x + 16 = 12x$

$4x = 16$

$x = 4$

$(x + 4)(x - 2) = 0$

$x = -4$ or $x = 2$

$\therefore$ ***After 6 (= 2 + 4) hours, Tom will be overtaken by Pete.***

$\therefore$ ***After Tome rode 48 (= 6 × 8) miles, he will be overtaken by Pete.***

(b) At what time will Tom and Pete be **8 miles** apart?

Let (A)hours be the time when Tom and Pete are 8 miles apart before Tom is overtaken by Pete.

$\therefore\ (8A + 16) - 12A = 8$

$-4A = -8$

$A = 2$

$\because$ ***Tom have already ridden for 2 hours.***

$\therefore$ ***Actual time when they are 8 miles apart is 4 (= 2 + 2) hours.***

Let (B)hours be the time when Tom and Pete are 8 miles apart after Tom is overtaken by Pete.

$\therefore\ 12B - 8B = 8$

$4B = 8$

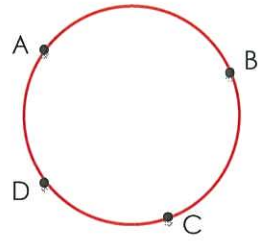
$B = 2$

$\because$ ***Pete already took 6 hours to overtake Tom.***

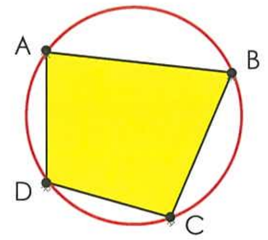
$\therefore$ ***Actual time when they are 8 miles apart is 8 (= 6 + 2) hours.***

13. Please read the following information and then do the proof on next page.

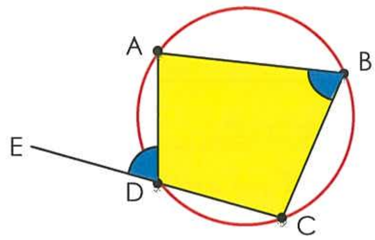
Points lie on the **same circle**, as the diagram below, are said to be **concyclic**. For example, A, B, C and D are **concyclic points**.



If the vertices of a **quadrilateral** lie on a **circle**, as the diagram below, then the quadrilateral is said to be **cyclic**. For example, ABCD is a **cyclic quadrilateral** since the vertices A, B, C and D lie on the circle.

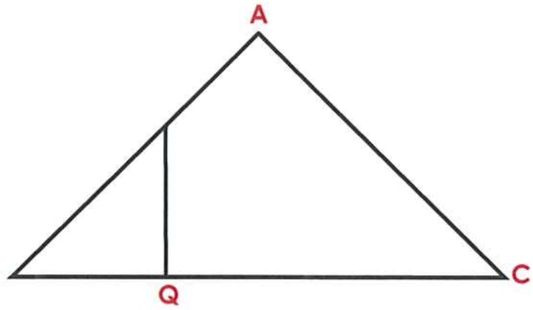


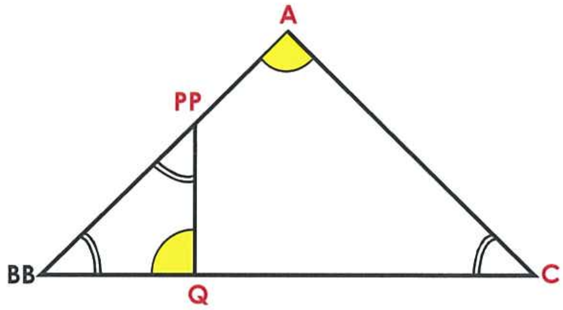
If the side CD is produced (i.e. extended) to E, as the diagram below, then **∠ADE** is called the **exterior angle of the cyclic quadrilateral** ABCD, and **∠ABC** is said to be the **interior opposite angle**.



Theorem: **If ∠ADE = ∠ABC, then A, B, C and D are concyclic. (ext.∠, int. opp.∠)**

In the figure below, **ΔABC** and **ΔBPQ** are **isosceles** triangles such that **AB = AC** and **BQ = PQ**.
Using the provided information about the concyclic points and cyclic quadrilateral, **prove** that **A, P, Q and C are concyclic**.





$\because\ AB = AC$

$\therefore\ \angle ABC = \angle ACB$ (base $\angle$s, isos. Δ)

$\because\ BQ = PQ$

$\therefore\ \angle QBP = \angle QPB$ (base $\angle$s, isos. Δ)

$\because\ \angle QBP = \angle ABC$ (common)

If $\angle ABC = x^\circ$, *then* $\angle ACB = \angle QBP = \angle QPB = x^\circ$

$\because\ \angle PQB = 180^\circ - \angle QBP - \angle QPB = 180^\circ - 2x$

$\angle BAC = 180^\circ - \angle ABC - \angle ACB = 180^\circ - 2x$

$\therefore\ \angle PQB = \angle BAC$

$\therefore$ *A, P, Q and C are concyclic.* (ext.$\angle$, int. opp.$\angle$)

End of Assessment